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The confining string beyond the free-string approximation in the gauge dual of percolation

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Abstract: We simulate five different systems belonging to the universality class of the gauge dual of three-dimensional random percolation to study the underlying effective string theory at finite temperature. All the data for the finite temperature string tension, when expressed by means of adimensional variables, are nicely described by a unique scaling function. We calculate the first few terms of the string tension up to order T^6 and compare to different theoretical predictions. We obtain unambiguous evidence that the coefficients of T^2 and T^4 terms coincide with those of the Nambu-Goto string, as expected, while the T^6 term strongly differs and is characteristic of the universality class of this specific gauge theory.

Keywords: Confinement, Lattice Gauge Field Theories, Bosonic Strings

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Contents

1 Introduction

The possibility of describing the long-distance dynamics of strong interactions in the confining phase by an effective string theory is a fascinating, many years old conjecture which dates from before the formulation of QCD [1]. In gauge theories it is based on the very intuitive assumption that the colour flux connecting a pair of distant quarks is concentrated, in the confining phase, inside a thin flux tube, which then generates the linear rising of the confining potential. According to the common lore, this thin flux tube should behave, when the quarks are pulled very far apart, as a free vibrating string [2].

The string-like nature of the flux tube is particularly evident in the strong coupling region, where the vacuum expectation value of large Wilson loops is given by a sum over certain lattice surfaces which can be considered as the world-sheets of the underlying confining string. At the roughening point [3] this sum diverges and the colour flux tube of whatever three-dimensional or four-dimensional lattice gauge theory undergoes a transition towards a rough phase. It is widely believed that such a phase transition of the flux tube belongs to the Kosterlitz-Thouless universality class [4]. Accordingly, the renormalisation group equations imply that the effective string action S describing the dynamics of the flux tube in the whole rough phase (the one connected with the continuum limit) flows at large scales towards a massless free field theory. Thus, for large enough inter-quark separations it is not necessary to know explicitly the specific form of the effective string action S, but only its infrared limit

$$S[h] = S_{cl} + S_0[h] + \dots , \tag{1.1}$$

where the classical action S_{cl} describes the usual perimeter-area term, h denotes the two-dimensional bosonic fields $h_i(\xi_1, \xi_2)$ with $i = 1, 2, \dots , d-2$ which describe the transverse displacements of the string with respect the configuration of minimal energy, ξ_1, ξ_2 are the coordinates on the world-sheet and $S_0[h]$ is the Gaussian action

$$S_0[h] = \frac{1}{2} \int d^2\xi \, \partial_\alpha h_i(\xi_1, \xi_2) \partial^\alpha h^i(\xi_1, \xi_2) \qquad (\alpha = 1, 2; \; i = 1, 2, \ldots, d-2) \; . \tag{1.2}$$

In this IR approximation the effective string is known as the free bosonic string. The ensuing universal string fluctuation effects [5, 6] were first unambiguously observed many years ago in the $\mathbb{Z}_2$ gauge theory in three dimensions [7, 8].

In order to study the first perturbative corrections to the IR limit it has been often assumed, for the sake of simplicity, that the effective string action is the Nambu-Goto action, i.e. the one proportional to the world-sheet area. Expanding in the natural dimensionless parameter $1/(\sigma A)$, where σ is the string tension and A the area of the minimal surface bounded by the Wilson loop, one can write

$$S[h] = S_{cl} + S_0[h] + \frac{1}{8\,\sigma A} \int d^2\xi \left[\left(\partial_\alpha h_i \partial^\alpha h^i \right)^2 - 2 \partial_\alpha h_i \partial^\beta h^i \partial^\alpha h_j \partial_\beta h^j \right] + O\left(\frac{1}{(\sigma A)^2} \right) \; . \tag{1.3}$$

Also in this case the first numerical analysis has been performed in a 3D $\mathbb{Z}_2$ gauge model [9]. More recently, high precision numerical simulations in SU(N) gauge theories confirmed these effects in the static quark potential [10, 11]. Mismatches between the observed spectrum of the low-lying string states with fixed ends and the predictions of the free bosonic and the Nambu-Goto strings have been repeatedly reported [12–15]. Recent studies demonstrated that these mismatches are gradually disappearing at larger distances [16, 17].

Closed strings wrapping around a compact dimension [18] or strings at finite temperature [19] have also been considered. In this case a remarkable agreement between the observed data and the Nambu-Goto predictions has been reported. From a theoretical point of view the reasons of this agreement can be understood, at least in part, resorting to a systematic expansion of the most general form of the effective string action $S[h]$ in terms of $h_\alpha(\xi)$ and its derivatives [10, 20]. The outcome of these studies can be conveniently summarised in some general properties of the first few terms of the low temperature expansion of the string tension

$$\sigma(T) = \sigma_0 - (d-2)\frac{\pi}{6} T^2 + \sum_{n \geq 3} s_n T^n \; . \tag{1.4}$$

The second term on the right hand side is the low temperature analogue of the Lüscher term of the inter-quark potential [21]. It is a characteristic quantum effect of the IR free string limit (1.1) and it is expected to be independent of the interaction terms of the effective theory. On the side of the gauge theory it is more than universal, in the sense that it does not depend on the nature of the gauge group.

As a consequence of a certain open-closed string duality [20] it was shown that for any number of space-time dimensions $s_3 \equiv 0$ and that in three dimensions s_4 is again a more than universal coefficient which can be evaluated in various ways [22, 23] and coincides with the Nambu-Goto value s_4^{NG}

$$s_4 = s_4^{\mathrm{NG}} = -(d-2)^2 \frac{\pi^2}{72\,\sigma_0} \; . \tag{1.5}$$

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A different approach to effective string theory [24] leads to similar conclusions [25, 26] (i.e. $s_3 = 0$, s_2 and s_4 more than universal), but for all values of d.

In spite of the remarkable agreement of the first few terms of the Nambu-Goto expansion with the numerical results, theoretical reasons indicate that the Nambu-Goto string is a sick theory and cannot describe the effective confining string to all orders in T: depending on the quantisation method, one finds either the breaking of rotational invariance or appearance of the conformal Liouville mode, in contrast with the assumption that the only physical degrees of freedom of the effective string are the transverse modes.

Numerical experiments lead to similar conclusions, showing that different gauge theories are described, at least at short distance, by different effective strings [14, 15], even if, so far, the order of the first term deviating from the more than universal behaviour in the power expansion of $\sigma(T)$ has not been determined. In this paper we find the order of such a term by evaluating in a particularly simple model, the gauge dual of random percolation in three dimensions, the coefficients s_n up to $n = 6$ order. We find that s_6 strongly deviates from the value predicted by the Nambu-Goto model.

We performed five different kinds of high-precision numerical experiments by varying the implementation of the percolation, the lattice spacing, the temporal extent of the lattice and the type of lattice. All these variations should keep the system in the same universality class. Indeed, as expected, all the collected data agree with the more than universal values of s_2 and s_4 and lead to

$$s_5 \simeq 0 \ ; \quad s_6 = \frac{\pi^3}{C\sigma_o^2} \, , \quad C \simeq 300 \ . \tag{1.6}$$

(See table 2 for more details). The vanishing of s_5 suggests that the high temperature expansion is even in T, like in Nambu-Goto case. Notice however that the value we find for s_6 for the gauge dual of percolation is very different from the corresponding coefficient of Nambu-Goto string, which is negative: $s_6^{\rm NG} = -(d-2)^3 \frac{\pi^3}{432\,\sigma_0^2}$. Preliminary results have been presented in [27, 28].

2 Polyakov loops

We focused on the behaviour of the Polyakov-Polyakov correlation function at finite temperature in a (2+1)-dimensional system. The lattice is a $L^2 \times \ell$ slice with periodic boundary conditions, with L large enough to represent the spatial extent and $\ell = \frac{1}{aT}$ the inverse temperature. We considered a pair of Polyakov loops orthogonal to the spatial direction and at a distance of r lattice spacings a; the (connected) correlation function in this case is denoted by $\langle\, P(0)P^*(r)\,\rangle$.

At the free string or leading order (LO) approximation (1.1) the functional form of this correlator in the effective string picture was calculated in different contexts. In lattice gauge theory it was first derived in [29], leading to

$$\langle\, P(0)P^*(r)\,\rangle_{\rm LO} \propto \frac{e^{-\sigma\ell r - \mu\ell}}{\eta(\tau)^{d-2}} \, , \tag{2.1}$$

where the Dedekind η function is defined as

$$\eta(\tau) \equiv q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n)\,, \quad \tau = \frac{i\ell}{2r}\,, \qquad q \equiv e^{2\pi i \tau}\ . \tag{2.2}$$

Within this approximation the temperature-dependent string tension, defined as the coefficient of the linear part of the confining potential, i. e.

$$\sigma(T) = -\lim_{r\to\infty} \frac{1}{rT} \log\langle P(0)P^*(r)\rangle\,, \tag{2.3}$$

turns out to be

$$\sigma(T) = \sigma - (d-2)\frac{\pi}{6}T^2 = \sigma_0 - (d-2)\frac{\pi}{6}T^2 + O(T^4)\,, \tag{2.4}$$

as expected from (1.4); σ_0 is the zero-temperature string tension and $\sigma = \sigma_0 + O(T^4)$.

At the next to the leading order (NLO) the functional form of the correlator has been calculated in [23]

$$\langle P(0)P^*(r)\rangle_{\text{NLO}} = \frac{e^{-\mu\ell - \tilde{\sigma} r\ell}}{\eta(\tau)^{d-2}} \left(1 + \frac{(d-2)\pi^2\ell[2E_4(\tau) + (d-4)E_2^2(\tau)]}{1152\tilde{\sigma} r^3} + O(\frac{1}{r^5})\right)\,, \tag{2.5}$$

where the functions E_2 and E_4 (second and fourth Eisenstein functions) are defined by:

$$E_2(\tau) \equiv 1 - 24\sum_{n=1}^{\infty} \sigma_1(n) q^n\,, \tag{2.6}$$

$$E_4(\tau) \equiv 1 + 240\sum_{n=1}^{\infty} \sigma_3(n) q^n\,. \tag{2.7}$$

The functions $\sigma_i(n)$ here represent the sum of the i-th powers of all divisors of n. Using the definition (2.3) one can easily verify that the parameter $\tilde{\sigma}$ is related to $\sigma(T)$ and σ_0 through

$$\sigma(T) = \tilde{\sigma} - \frac{\pi}{6}T^2 - \frac{\pi^2}{72\tilde{\sigma}}T^4 = \sigma_0 - \frac{\pi}{6}T^2 - \frac{\pi^2}{72\sigma_0}T^4 + O(T^5)\ . \tag{2.8}$$

3 The model

In this work, the model we chose as laboratory to study the effective string theory is the three-dimensional random percolation [30]. It can be seen as the gauge dual of a Q-state Potts model in the limit $Q \to 1$.

It is well known that for integer $Q > 1$ one can formulate the gauge Potts model either in terms of gauge fields or in the dual version in terms of the spin variables. From a computational point of view the latter is much more convenient in lattice simulations. It is then useful to map the needed gauge invariant observables (Wilson loops or Polyakov correlators) into the corresponding quantities of the dual version and not to worry about the gauge formulation. This approach particularly well suits random percolation, as the direct gauge formulation is not (yet) known, but the rules to evaluate the gauge invariant observables are unambiguously defined in any configuration of the system, as we shall see below.

In the bond percolation model the lattice configurations are generated as follows. Each link of a three-dimensional lattice Λ is independently set to *on* or *off* according to some fixed probability p, which plays the role of a coupling constant. The set of *on* links, or *active links*, forms a graph G, whose connected components are known as clusters. When p exceeds a threshold value p_c, depending on the nature of the lattice, an infinite, percolating cluster forms.

Similarly, in the site percolation model, another possible formulation of the random percolation, one independently sets *on* or *off* the nodes of the lattice with a fixed probability p and generates a graph G by putting an active link for each pair of adjacent *on* nodes.

The key ingredient to extract from the above ensemble of graphs the relevant information on the underlying gauge theory is the definition of the percolation counterpart of the Wilson operator W_γ associated with whatever closed path of the dual lattice. We set $W_\gamma(G) = 1$ if there is no path of G topologically linked to γ, otherwise we set $W_\gamma(G) = 0$. In other words, W_γ is a projector on the ensemble of graphs whose image is the subset of graphs not linked to γ. Therefore its vacuum expectation value $\langle W_\gamma \rangle$ coincides with the average probability that there is no path in any cluster linked to γ.

As in usual gauge theories, evaluating these quantities yields the main physical properties of the model. In this way it has been shown that the percolating phase is confining. The string tension σ and the other physical observables have the expected scaling behaviour dictated by the universality class of three-dimensional percolation, therefore such a theory has a well-defined continuum limit [30]. Moreover it has a non-trivial glueball spectrum [31] and a second-order deconfining transition at finite temperature T_c with a ratio $T_c/\sqrt{\sigma} \simeq 1.5$ which turns out to be universal, i.e. it does not depend on the kind of lattice utilised nor on the specific percolation process considered (bond or site percolation).

In the study of the finite size effects described by the effective string theory one could use in principle whatever confining gauge theory, owing to the fact that the dominant effects do not depend on the gauge group. The great advantage of the dual percolation model we study in this paper is that its simplicity allows to explore regions that are still inaccessible to the other gauge systems from a computational point of view.

4 Methodology

In this section we describe the principal aspects of our method, based on the direct measurement of the correlator of two coplanar Polyakov loops at finite $T < T_c$. We begin with the description of the lattice algorithm and proceed to discuss the kind of fits we use to extract the temperature-dependent string tension.

4.1 Simulations

We are interested in the universal properties of the effective string theory in the gauge dual of percolation, therefore we studied how the system responds to a variation of the spatial and the temporal sizes of the lattices, of the occupancy probability p, of the kind of

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Lattice	p	$\ell_c = 1/aT_c$	temporal sizes ℓ	spatial sizes
SC bond	0.272380	6	$9 \div 15$	128
SC bond	0.268459	7	$10 \div 15$	128
SC bond	0.265615	8	$10 \div 17$	128;194;256;320
SC site	0.3459514	7	$11 \div 17$	128
BCC bond	0.21113018	3	$4 \div 10$	128

Table 1. Relevant parameters of the simulations.

percolation (bond or site) and finally of the geometry of the lattice, considering both the simple cubic lattice (SC) and the body-centred cubic lattice (BCC). The set of simulations is listed on table 1.

The values of p are taken from [30] and are the occupancy probabilities corresponding to systems which are at the deconfining temperature $aT_c = 1/\ell_c$ when the (periodic) temporal extension in units of lattice spacing a is the value ℓ_c reported in the table. The simulations were made in the confined phase with a temporal extension in the range $\ell_c < \ell \lesssim 3\ell_c$ where the Polyakov-Polyakov correlator is well described by the NLO formula (2.5). The spatial size was 128^2 which was in most cases amply sufficient to account for the infinite volume limit. Only in two cases, namely $\ell = 10$ and $\ell = 11$ with $\ell_c = 8$, we observed a non-negligible dependence on the spatial size. In those cases we performed further simulations on larger lattices, as indicated on the table, and extracted the corrected value of the string tension $\tilde{\sigma}$ using the scaling relation

$$\tilde{\sigma}_{1/L} = \tilde{\sigma} - c\, L^{-1/\nu_2}\,, \tag{4.1}$$

where $\nu_2 = \frac{4}{3}$ is the thermal exponent of two-dimensional random percolation. In both cases the fit to the data was very good.

To reach an acceptable statistics, we collected data from 10^5 configurations for each value of p and ℓ.

4.2 Algorithm

Due to the particular nature of the random percolation model, each configuration can be generated independently from scratch, by simply filling an empty lattice with links (or sites) that are randomly switched on with a probability p.

The tricky part is the measurement of the topological linking of the resulting graph G with a pair of Polyakov loops; to this end, we first choose a cylindric surface Σ bounded by the two loops and look for the closed paths of G intersecting it and linked with one of the two loops. It is convenient to "clean up" the graph $G \to G'$, getting rid of dead ends and bridges between loops, as they cannot belong to the mentioned closed paths [30]. This is done once for the whole configuration.

On this "minimal" configuration G', then, the surface Σ is translated in all possible positions and the linking is measured with the technique of reconstructing each time the clusters in the configuration (by means of the Hoshen-Kopelmann algorithm) keeping track of the crossings of the loop surface, in order to detect nonzero winding numbers.

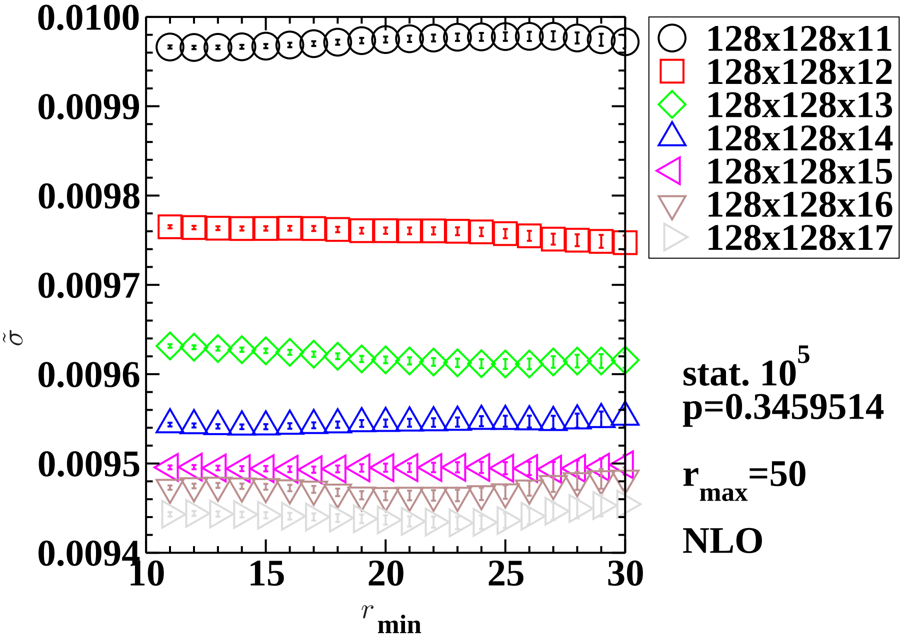

Figure 1. The fitted value of $\tilde{\sigma}$ to (2.5) as a function of the minimal distance $r_{\rm min}$ of the set of Polyakov-Polyakov correlators considered in the fit for the site percolation in SC lattice with $\ell_c = 7$. The different plateaux correspond to different values of the temporal extension ℓ. A similar plot for the bond percolation in the SC lattice can be found in [27] for $\ell_c = 6$ and in [28] for $\ell_c = 8$.

4.3 Fits

The measured Polyakov-Polyakov correlators are compared with the expected behaviour (2.5). Being this an asymptotic expression, valid in the IR limit, we fitted the data to (2.5) by progressively discarding the short distance correlators and taking all the values in the range $r_{\rm min} \leq r \leq r_{\rm max} = 50a$, with $r_{\rm min}$ varying from the value ℓ indicated in the table 1 to 40 lattice spacings a. The value of the fitted parameter $\tilde{\sigma}$ as a function of $r_{\rm min}$ is plotted in figure 1. The large plateaux in the whole range of the temporal extension ℓ considered show the stability of the fit which is also supported by a χ^2/dof of the order of 1 or less. In some cases, when ℓ is too close to ℓ_c, the plateau starts at larger values of $r_{\rm min}$ and correspondingly the χ^2 test is not good. We discarded these data from the further analysis. In all other cases the Polyakov-Polyakov correlator in the examined range of r and l is well described by the asymptotic formula (2.5). Since the latter is a result of the continuum, this agreement can also be interpreted as a check for the absence of finite lattice spacing effects at the level of our statistical accuracy.

It is important to note that the fitted parameter $\tilde{\sigma}$ is not yet the string tension at zero temperature σ_0, since (2.5) is not an exact formula, but only takes into account the temperature dependence up to the order T^4. On general grounds we expect

$$\tilde{\sigma} = \sigma_0 + O(T^5) \ . \tag{4.2}$$

If it turned out that the dependence of the parameter $\tilde{\sigma}$ on T involved lower powers of T, i.e. T^2 and/or T^4, it would mean that the first two thermal corrections in (2.8) were not

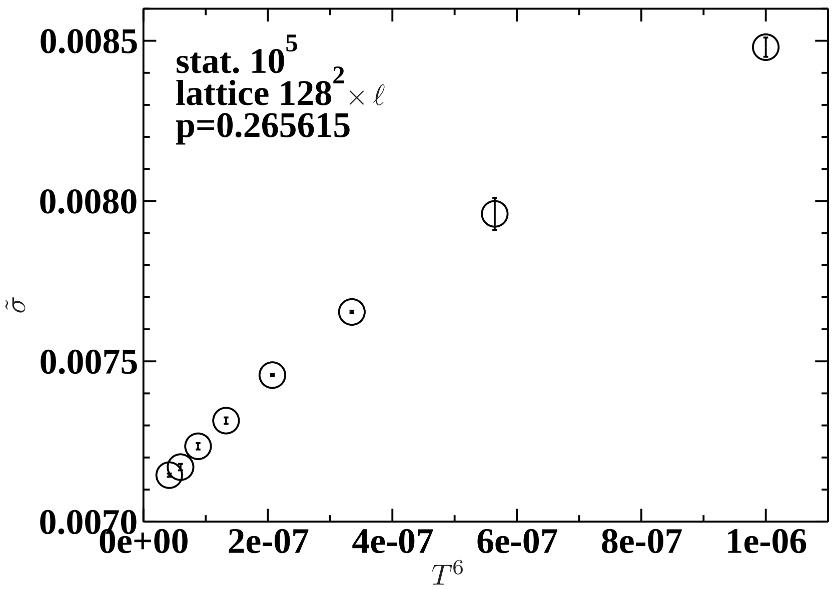

Figure 2. Plot of the fitting parameter $\tilde{\sigma}$ as a function of T^6 in numerical experiments with bond percolation with $\ell_c = 8$. A similar plot for the case $\ell_c = 7$ can be found in [28].

universal. This question can be settled by studying the dependence on ℓ of the mentioned plateaux. In all the cases it turns out that for $aT = 1/\ell$ low enough the correction is proportional to T^6 (see for instance figure 2). We inserted the fitted parameter $\tilde{\sigma}$ in (2.8) in order to reconstruct the quantity $\sigma(T)$ for the whole set of temperatures listed in the fourth column of table 1. We then performed, for each line of such a table, a two-parameter fit to the formula

$$\sigma(T) = \sigma_0 - \frac{\pi}{6}T^2 - \frac{\pi^2}{72\sigma_0}T^4 + \frac{\pi^3}{C\sigma_0^2}T^6 + O(T^8)\ . \tag{4.3}$$

The fitted parameters σ_0 and C turn out to be stable. Their values are reported in table 2. Another way to analyze the data is to combine (4.2) with the observation that the term T^5 is absent and fit directly the parameter $\tilde{\sigma}$ to the formula $\tilde{\sigma} = \sigma_0 + \frac{\pi^3}{C\sigma_0^2}T^6$. This way of analysing the data differs from the previous one for terms of the order $O(T^8)$, thus it can be used for a rough estimate of the systematic errors. It turns out that the evaluations of σ_0 coincide, within the statistical errors, with the values determined in the other way, while the estimates of C are about 10% larger than the values reported in table 2.

5 Results and conclusion

In this paper we combined Monte Carlo simulations with different finite-size scaling techniques applied to various percolating systems. The outcome of the extensive numerical experiments on the gauge dual of random percolation and the analysis described in the previous section is a precise determination of the string tension as a function of the temperature in a wide range of T.

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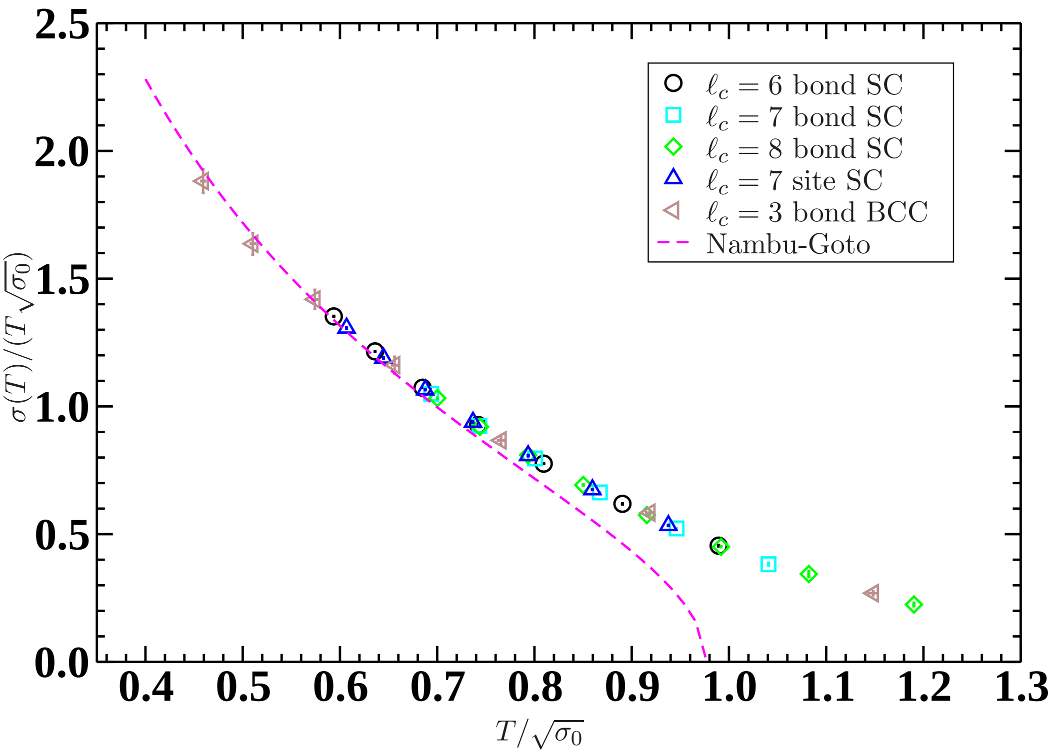

Figure 3. Plot of the scaling variable $\sigma(T)/(T\sqrt{\sigma_0})$ as a function of $T/\sqrt{\sigma_0}$. The dashed line is the result of the Nambu-Goto string.

If we plot the adimensional ratio $\sigma(T)/(T\sqrt{\sigma_0})$ versus the adimensional temperature $T/\sqrt{\sigma_0}$ it turns out that all the data neatly lie on a unique universal curve as figure 3 shows. This scaling behaviour indicates that the most relevant sources of systematic errors, including the approach to the infinite volume and the continuum limits, have been taken into account. The plotted quantity is expected to vanish at T_c with the power law $\sim (T_c - T)^{\nu_2}$, where $\nu_2 = \frac{4}{3}$ is the thermal exponent of 2D percolation. Unfortunately our data are not sufficiently close to T_c in order to check accurately this behaviour. A similar scaling function has been determined for the 3D SU(2) gauge model [32].

From each set of the numerical simulations described in each row of table 1 we can extract three physical quantities. The first one is the coefficient C of eq. (4.3) which determines the T^6 correction to the string tension. The five values of C generated by as many different systems (see table 2) remarkably coincide up to the statistical errors. Each set of simulations yields also a precise determination of $a^2\sigma_0$ which, combined with the precise value of the deconfinement temperature in the same lattice units, yields the adimensional ratio $T_c/\sqrt{\sigma_0}$. This quantity is expected to be constant in the continuum limit.

In order to extrapolate to this limit, one has to take into account the correction to scaling terms. The string tension in the gauge dual of percolation is expected to obey the scaling behaviour [30]

$$a^2\sigma(p) = S(p-p_c)^{2\nu}\left(\frac{1}{1+B(p-p_c)^{\omega\nu}}\right), \tag{5.1}$$

Lattice	$\ell_c = 1/aT_c$	C	$a^2\sigma_0$	χ^2/dof	$T_c/\sqrt{\sigma_0}$
SC bond	6	291(7)	0.012612(6)	0.15	1.4841(4)
SC bond	7	281(5)	0.009234(5)	1.2	1.4866(5)
SC bond	8	297(5)	0.007059(5)	0.4	1.4878(5)
SC site	7	307(9)	0.009399(8)	0.2	1.4735(6)
BCC bond	3	295(14)	0.0474(4)	0.8	1.531(7)

Table 2. The parameter C and $a^2\sigma_0$ in the fit (4.3) and χ^2/dof which are obtained for the corresponding numerical experiments listed in table 1. The last column is the universal ratio $T_c/\sqrt{\sigma_0}$ as obtained by combining the second and the fourth columns.

where p_c is the critical threshold and ν and ω are the thermal and correction-to-scaling exponents of 3D percolation (see [33] for an accurate numerical estimate of these exponents). Similarly, the deconfining temperature T_c is expected to scale as

$$a\,T_c = \mathcal{T}(p-p_c)^\nu \left(\frac{1}{1+C(p-p_c)^{\omega\nu}}\right) \;. \tag{5.2}$$

When applied to the case of bond percolation in the SC lattice they yield $S = 9.29(2)$ and $\mathcal{T} = 4.562(1)$, thus the extrapolated continuum limit of $T_c/\sqrt{\sigma_0}$ is estimated to be $\mathcal{T}/\sqrt{S} = 1.497(2)$.

In conclusion, in this paper we extracted from various three-dimensional percolating systems some general information on the effective string theory describing the infrared properties of the confining phase of the gauge dual of percolation at finite temperature. We numerically evaluated the universal scaling function describing the string tension as a function of the temperature. We obtained clear evidence that the first two non-vanishing coefficients of the expansion of σ in powers of T coincide with those of the Nambu-Goto string, while the third one strongly differs. Nonetheless this term does not depend on the UV cut-off nor on the specific percolation model, but is characteristic of the universality class of (the gauge dual of) the three-dimensional random percolation.

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